All questions must be answered. Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

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# MONETARY ECONOMICS: MACRO ASPECTS
# SOLUTIONS TO AUGUST 15, 2014 EXAM

## QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) The Friedman rule for optimal monetary conduct implies that the central bank should target zero inflation.

A *False.* The Friedman rule states that the private opportunity cost of money should equal the public, which is (setting aside production costs) zero. As the private opportunity cost of holding money is the nominal interest rate, this should be zero under the Friedman rule. In terms of price developments, one uses that the Fisher equation approximately links the nominal and real interest rate as $i = r + \pi$, where $i$ is the nominal interest rate, $r$ is the real interest rate and $\pi$ is the inflation rate. The Friedman rule of $i = 0$ thus implies that $\pi = -r$, i.e., inflation should equal the negative of the real interest rate.

(ii) Under optimal inflation targeting, a positive coefficient on the output gap in the associated interest-rate rule implies that the central bank has preferences for stable output.

A *False.* Per se, coefficients in the optimal interest-rate rule under inflation targeting cannot give information about policy preferences. In the curriculum we have seen examples of "strict" inflation targeting, i.e., cases where the central bank only cares anout inflation stability, where optimal policy involves a response to the output gap. This is the case when the output gap is informative about inflation developments. The output gap is in such a case an intermediate target for monetary policymaking.

(iii) In the simple New-Keynesian model, a history-dependent monetary policy is disadvantageous.

A *False.* In the New-Keynesian model with variables being affected by forward-looking expectations, there will often be a difference between ex ante and ex post optimal monetary policy. I.e., policies that are optimal ex ante may not be time consistent. This is because it is optimal through policy to influence expectations about the future so as to affect current outcomes in a desirable way. One way to design such policies is through a commitment to history dependence: By letting policy credibly depend on the past, one is able to affect expectations about the future through current actions (today is tomorrow's past).
An example from the curriculum is the case of a temporary inflationary shock. This can be stabilized to some extent by contractionary monetary policy at the cost of a loss of output. With a history-dependent policy, the policymaker promises to continue the contraction into the future—even after the shock is vanished. This reduces inflation expectations, and thus current inflation, which implies that a smaller output loss is required. So, history dependence is advantageous (albeit not time consistent).

## QUESTION 2:

### Nominal and real rigidities and monetary policy

Consider an economy with time dependent, staggered price setting, where in any period half of intermediate goods producers sets a price, which is fixed for two periods. Let $\overline{p}_{t+j}$ be the log of prices fixed for periods $t+j$ and $t+j+1$, and note that aggregate prices $p_t$ are given by $p_t = \frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\overline{p}_t$.

(i) It can be shown that profit maximization leads to the following pricing rule for firms resetting prices in period $t$:

$$\overline{p}_t = \frac{1}{2}\left(p_t + \mathrm{E}_t p_{t+1}\right) + \frac{1}{2}\left(v_t + \mathrm{E}_t v_{t+1}\right), \tag{1}$$

where $v_t$ is log of real marginal costs and $\mathrm{E}_t$ is the rational expectations operator. Explain the economics behind (1).

A In this variant of the Taylor model, firms would absent any restrictions on price setting set prices as a markup over nominal marginal costs in each period. Put differently, they would choose a real price proportional to their real marginal costs. However, when firms face the restriction that the price they set are fixed for two periods, it will optimally chose the price in period $t$, $\overline{p}_t$, such that its desired expected average price over the two periods the price is fixed will be proportional to the average of current and future real marginal costs. This is what (1) depicts. It is crucial to note the forward-looking nature of individual firms' price setting when there is nominal rigidity,

Assume that real marginal costs are linearly related to real output $y_t$,

$$v_t = \gamma y_t, \qquad 1 \geq \gamma > 0, \tag{2}$$

and that aggregate demand can be characterized by

$$m_t - p_t = y_t, \tag{3}$$

where $m_t$ is log of the nominal money supply. The money supply is assumed to follow a random walk, i.e., $\mathrm{E}_t m_{t+1} = m_t$.

(ii) Show that (1), (2) and (3) along with the definition of $p_t$ can be solved for the following pricing rule:

$$\overline{p}_t = a\overline{p}_{t-1} + (1-a)\, m_t, \qquad a = \frac{1-\sqrt{\gamma}}{1+\sqrt{\gamma}}. \tag{4}$$

[Hint: Rewrite (1) as a second-order difference equation in $\overline{p}_t$ as a function of $m_t$ only, and solve it by the method of undetermined coefficients by conjecturing $\overline{p}_t = a\overline{p}_{t-1} + (1-a)\, m_t$.]

A Insert $p_t = \frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\overline{p}_t$ into (1) to get

$$\begin{aligned}
\overline{p}_t &= \frac{1}{2}\left(\frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\overline{p}_t + \mathrm{E}_t\left[\frac{1}{2}\overline{p}_t + \frac{1}{2}\overline{p}_{t+1}\right]\right) + \frac{1}{2}\left(v_t + \mathrm{E}_t v_{t+1}\right), \\
2\overline{p}_t &= \frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\overline{p}_t + \mathrm{E}_t\left[\frac{1}{2}\overline{p}_t + \frac{1}{2}\overline{p}_{t+1}\right] + v_t + \mathrm{E}_t v_{t+1}, \\
\overline{p}_t &= \frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\mathrm{E}_t\overline{p}_{t+1} + v_t + \mathrm{E}_t v_{t+1}.
\end{aligned}$$

Then use (2) and (3) to substitute out $v_t$ and $y_t$:

$$\overline{p}_t = \frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\mathrm{E}_t\overline{p}_{t+1} + \gamma\left(m_t - p_t + \mathrm{E}_t\left[m_{t+1} - p_{t+1}\right]\right).$$

Use the assumption that $m_t$ follows a random walk to get

$$\overline{p}_t = \frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\mathrm{E}_t\overline{p}_{t+1} + \gamma\left(2m_t - p_t - \mathrm{E}_t p_{t+1}\right).$$

Then use $p_t = \frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\overline{p}_t$ to get:

$$\overline{p}_t = \frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\mathrm{E}_t\overline{p}_{t+1} + \gamma\left[2m_t - \frac{1}{2}\left(\overline{p}_{t-1} + \overline{p}_t\right) - \mathrm{E}_t\frac{1}{2}\left(\overline{p}_t + \overline{p}_{t+1}\right)\right],$$

which solved for $\overline{p}_t$:

$$\overline{p}_t\left(1 + \frac{\gamma}{2} + \frac{\gamma}{2}\right) = \frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\mathrm{E}_t\overline{p}_{t+1} + \gamma\left[2m_t - \frac{1}{2}\overline{p}_{t-1} - \mathrm{E}_t\frac{1}{2}\overline{p}_{t+1}\right].$$

This immediately yields

$$\overline{p}_t = \frac{1}{2}\frac{1-\gamma}{1+\gamma}\left[\overline{p}_{t-1} + \mathrm{E}_t\overline{p}_{t+1}\right] + \frac{2\gamma}{1+\gamma}m_t. \tag{*}$$

To solve (*), follow the hint and conjecture a solution of the form

$$\overline{p}_t = a\overline{p}_{t-1} + (1-a)\,m_t,$$

where $a$ is a coefficient to be determined. Forward the conjecture one period and take period-$t$ expectations (and use again that $m_t$ follows a random walk):

$$\mathrm{E}_t\overline{p}_{t+1} = a\overline{p}_t + (1-a)\,m_t,$$

Insert this into (*):

$$\overline{p}_t = \frac{1}{2}\frac{1-\gamma}{1+\gamma}\left[\overline{p}_{t-1} + a\overline{p}_t + (1-a)\,m_t\right] + \frac{2\gamma}{1+\gamma}m_t,$$

and solve for $\overline{p}_t$:

$$\overline{p}_t\left[1 - \frac{a}{2}\frac{1-\gamma}{1+\gamma}\right] = \frac{1}{2}\frac{1-\gamma}{1+\gamma}\left[\overline{p}_{t-1} + (1-a)\,m_t\right] + \frac{2\gamma}{1+\gamma}m_t,$$

$$\overline{p}_t = \frac{1-\gamma}{2\,(1+\gamma) - a\,(1-\gamma)}\overline{p}_{t-1} + \frac{(1-\gamma)\,(1-a) + 4\gamma}{2\,(1+\gamma) - a\,(1-\gamma)}m_t.$$

This verifies the form of the conjecture, and shows that the undetermined coefficient must satisfy[1]

$$a = \frac{1-\gamma}{2\,(1+\gamma) - a\,(1-\gamma)}. \tag{**}$$

Hence,

$$-a^2\,(1-\gamma) + 2\,(1+\gamma)\,a - (1-\gamma) = 0,$$

[1]It must also satisfy

$$1 - a = \frac{(1-\gamma)\,(1-a) + 4\gamma}{2\,(1+\gamma) - a\,(1-\gamma)}.$$

which shows that $a$ has two solutions given by

$$\begin{aligned} a &= \frac{2(1+\gamma) \pm \sqrt{4(1+\gamma)^2 - 4(1-\gamma)^2}}{2(1-\gamma)} \\ &= \frac{1+\gamma \pm \sqrt{(1+\gamma)^2 - (1-\gamma)^2}}{1-\gamma} \\ &= \frac{1+\gamma \pm 2\sqrt{\gamma}}{1-\gamma}. \end{aligned}$$

To provide a stable solution ($a < 1$), the lower root is the relevant. I.e.,

$$a = \frac{1+\gamma - 2\sqrt{\gamma}}{1-\gamma}$$

This can be rewritten as

$$a = \frac{\left(1-\sqrt{\gamma}\right)^2}{\left(1-\sqrt{\gamma}\right)\left(1+\sqrt{\gamma}\right)} = \frac{1-\sqrt{\gamma}}{1+\sqrt{\gamma}},$$

as stated in the hint.

(iii) Interpret (4) and explain how the persistence of monetary shocks depends on the degree of real rigidity (here interpreted as the inverse of $\gamma$).

A From the solution we see that $a > 0$ as $\gamma < 1$, implying that price adjustment is gradual. Hence, the real effects of a nominal money shock are persistent. This means that even after *all* firms have had the opportunity to adjust prices, aggregate price adjustment is incomplete, and output has not returned to steady state. The reason is that those adjusting in period $t$, do not adjust fully to a monetary shock as their real marginal cost does not rise sufficiently (full adjustment only happens if $\gamma = 1$). The smaller is $\gamma$, i.e., the higher real rigidity,

But this can be rewritten as

$$\begin{aligned} a &= 1 - \frac{(1-\gamma)(1-a) + 4\gamma}{2(1+\gamma) - a(1-\gamma)} \\ &= \frac{2(1+\gamma) - a(1-\gamma) - (1-\gamma)(1-a) - 4\gamma}{2(1+\gamma) - a(1-\gamma)} \\ &= \frac{2(1+\gamma) - (1-\gamma) - 4\gamma}{2(1+\gamma) - a(1-\gamma)} \\ &= \frac{1-\gamma}{2(1+\gamma) - a(1-\gamma)}, \end{aligned}$$

which is the same as (**). Showing the validity of the conjecture's restriction that the coefficents on $\overline{p}_{t-1}$ and $m_t$ sum to one is not required.

the less they adjust. Hence, aggregate prices will adjust by less than half of the change in the money supply. This feeds into next period's price adjustment, where firms adjusting prices will also dampen their adjustment.
As this is foreseen by current price setters, it further dampens current price adjustment. Hence, the interplay between backward and forward-looking elements in price setting creates persistence, which is greater the less sensitive real marginal costs are to output (firms' demand). Hence, lower $\gamma$, higher real rigidity, gives less nominal adjustment and thereby longer lasting effects of nominal money shocks.

## QUESTION 3:

**Cash-in-advance and optimal monetary policy**

Consider a flex-price economy with a cash-in-advance constraint on consumption purchases. Life-time utility of the representative household is

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1,$$

where $c_t$ is consumption in period $t$ and the function $u$ satisfies $u' > 0$, $u'' < 0$. Households satisfy the budget constraint

$$\begin{aligned} \omega_t &\equiv f(k_{t-1}) + \tau_t + (1-\delta) k_{t-1} + \frac{m_{t-1} + (1+i_{t-1}) b_{t-1}}{1+\pi_t} \\ &= c_t + k_t + m_t + b_t, \end{aligned} \tag{1}$$

and the cash-in-advance constraint

$$c_t \leq \frac{m_{t-1}}{1+\pi_t} + \tau_t. \tag{2}$$

In (1) and (2), $k_{t-1}$ is physical capital at the end of period $t-1$, $f$ is a production function with $f' > 0$, $f'' < 0$, $\tau_t$ are real government monetary transfers, $0 < \delta < 1$ is the depreciation rate, $m_{t-1}$ is real money balances at the end of period $t-1$, $\pi_t$ is the inflation rate, $i_{t-1}$ is the nominal interest rate on bonds, $b_{t-1}$ is the real stock of bonds at the end of period $t-1$.

(i) Discuss the model and explain the constraints (1) and (2).

A This discussion should go through the elements of the budget constraint (1), and mention should be given to how inflation erodes the real value of money holdings. In the discussion of (2), the weak inequality should be explained. Also, it is fine to mention that (2) is a variant of the CIA constraint where the goods market "opens" before the financial markets.

(ii) Let the value function $V$ be defined by

$$V(\omega_t, m_{t-1}) = \max_{c_t, k_t, m_t} \left\{ u(c_t) + \beta V(\omega_{t+1}, m_t) - \mu_t \left( c_t - \frac{m_{t-1}}{1+\pi_t} - \tau_t \right) \right\},$$

where

$$\omega_{t+1} = f(k_t) + \tau_{t+1} + (1-\delta) k_t + \frac{m_t}{1+\pi_{t+1}} + (1+r_t)(\omega_t - c_t - k_t - m_t)$$

follows by (1) with $1+r_t \equiv (1+i_t)/(1+\pi_{t+1})$, and where $\mu_t$ is the Lagrange multiplier on (2). Show by dynamic programming that optimal behavior results in

$$i_t = \frac{\mu_{t+1}}{V_\omega(\omega_{t+1}, m_t)}.$$

Interpret this condition economically.

A The first-order conditions with respect to $c_t$, $k_t$ and $m_t$ are

$$u_c(c_t) = \beta(1+r_t) V_\omega(\omega_{t+1}, m_t) + \mu_t$$

$$\beta V_\omega(\omega_{t+1}, m_t)[f_k(k_t) + 1 - \delta] = \beta(1+r_t) V_\omega(\omega_{t+1}, m_t)$$

$$\beta \frac{1}{1+\pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) = \beta(1+r_t) V_\omega(\omega_{t+1}, m_t)$$

Taking the partial derivatives of the value function with respect to $\omega_t$ and $m_{t-1}$, and applying the envelope theorem, gives

$$V_\omega(\omega_t, m_{t-1}) = \beta(1+r_t) V_\omega(\omega_{t+1}, m_t), \tag{*}$$

$$V_m(\omega_t, m_{t-1}) = \mu_t \frac{1}{1+\pi_t}. \tag{**}$$

One can then use (**) forwarded one period to substiute out $V_m(\omega_{t+1}, m_t)$ from the first-order condition with respect to $m_t$:

$$\beta \frac{1}{1+\pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta \mu_{t+1} \frac{1}{1+\pi_{t+1}} = \beta(1+r_t) V_\omega(\omega_{t+1}, m_t).$$

Using the definition of $r_t$, this becomes

$$\begin{aligned} \beta \frac{1}{1+\pi_{t+1}} V_\omega (\omega_{t+1}, m_t) + \beta \mu_{t+1} \frac{1}{1+\pi_{t+1}} &= \beta \frac{1+i_t}{1+\pi_{t+1}} V_\omega (\omega_{t+1}, m_t), \\ V_\omega (\omega_{t+1}, m_t) + \mu_{t+1} &= (1+i_t) V_\omega (\omega_{t+1}, m_t), \end{aligned}$$

and thus

$$i_t = \frac{\mu_{t+1}}{V_\omega (\omega_{t+1}, m_t)}$$

as required. This mainly shows that the nominal interest rate is positive when money has liquidity services, i.e., when the CIA constraint binds, $\mu_{t+1} > 0$. In other words, it cannot be optimal to hold more money that just necessary to carry out consumption purchases when the nominal interest rate—the opportunity cost of real money holdings—is positive.

(iii) Does this economy exhibit superneutrality in steady state? What is the optimal nominal interest rate? Explain.

A From the steady-state condition arising from the first-order condition with respect to $k_t$ one gets:

$$f_k (k^{ss}) + 1 - \delta = 1 + r^{ss},$$

and from (*) one gets

$$1 = \beta (1 + r^{ss}),$$

implying

$$f_k (k^{ss}) + 1 - \delta = 1/\beta.$$

Hence, long-run capital and output per capita are neutral w.r.t. monetary factors. Steady-state consumption follows from the national account as

$$c^{ss} = f (k^{ss}) - \delta k^{ss}$$

I.e., long-run superneutrality holds. As utility only depends on consumption, monetary policy has no impact on welfare. Hence, *any* nominal interest rate is optimal.